4/14/16

HW: "Arithmetic and Geometric Sequences" homework section #
Test 1 on Thursday 4/21

AIM: What are Arithmetic and Geometric Sequences?

Add

ARITHMETIC SEQUENCE RECURSIVE DEFINITION

Given $f(1)$, then $f(n)=f(n-1)+d$ or given a_1 then $a_n = a_{n-1}+d$

where d is called the **common difference** and can be positive or negative.

"d" is what is added to get the next term.

***Exercise* #1:** Generate the next three terms of the given arithmetic sequences.

common difference

(a) $f(n) = f(n-1) + 6$ with $f(1) = 2$

difference

Start

term wanted

previous term

$f(1) = 2$

$f(2) = 2 + 6 = 8$

$f(3) = 8 + 6 = 14$

$f(4) = 14 + 6 = 20$

(b) $a_n = a_{n-1} + \frac{1}{2}$ and $a_1 = \frac{3}{2}$

Start

term wanted

previous

$a_1 = \frac{3}{2}$

$a_2 = \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2$

$a_3 = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$

$a_4 = \frac{5}{2} + \frac{1}{2} = \frac{6}{2} = 3$

$2, \frac{5}{2}, 3$

$2, 2.5, 3$

***Exercise* #2:** For some number t, the first three terms of an ~~arithmetic sequence~~ are $2t$, $5t-1$, and $6t+2$. What is the ~~numerical value~~ of the fourth term? Hint: first set up an equation that will solve for t.

$2t$ (4), $5t-1$ (9), $6t+2$ (14), $9t+1$

$t=2$ $\quad 9(2)+1 = 19$

$d = (5t-1) - 2t = 3t - 1$

$d = (6t+2) - (5t-1) = t + 3$

Same b/c the "d" is consistent

$t + 3 = 3t - 1$

$-t + 1 \quad -t + 1$

$4 = 2t$

$2 = t$

3 6 9 12 ___

$\xrightarrow{\text{Add}}$

$\xleftarrow{\text{Sub}}$

Exercise #3: Consider $a_n = a_{n-1} + 3$ with $a_1 = 5$.

$d = 3$

(a) Determine the value of a_2, a_3, and a_4.

5	8	11	14
a_1	a_2	a_3	a_4
	$5+3=8$	$8+3$	$11+3$

(b) How many times was 3 added to 5 in order to produce a_4?

3 times (1 less than # of term)

(c) Use your result from part (b) to quickly find the value of a_{50}.

$5 + 3(50-1)$

$5 + 3(49)$

$5 + 147 = 152$

$a_{50} = 152$

(d) Write a formula for the n^{th} term of an arithmetic sequence, a_n, based on the first term, a_1, d and n.

n^{th} term = first term + common difference $(n-1)$

$$\boxed{a_n = a_1 + d(n-1)}$$

General Formula to find any term in Arithmetic Sequence.

HW: Homework #1-6
(Fluency)

add

Exercise #4: Given that $a_1 = 6$ and $a_4 = 18$ are members of an arithmetic sequence, determine the value of a_{20}.

$a_n = a_1 + d(n-1)$

Term now

First Term

common difference

number of term now.

Given:

6			18
a_1	a_2	a_3	a_4

1 2 3

$18-6=12$ ← total added

$\frac{12}{3}=4$ ← difference each time.

Have

$a_1 = 6$

$d = 4$

$n = 20$

$a_{20} = 6 + 4(20-1)$

$a_{20} = 6 + 76$

$a_{20} = \boxed{82}$

GEOMETRIC SEQUENCE RECURSIVE DEFINITION

Given $f(1)$ then $f(n)=f(n-1)\cdot r$ or given a_1, then $a_n=a_{n-1}\cdot r$

where r is called the **common ratio** and can be positive or negative and is often fractional.

***Exercise* #5:** Generate the next three terms of the geometric sequences given below.

(a) $a_1=4$ and $r=2$

$a_1=4$
$a_2=4(2)=\boxed{8}$
$a_3=8(2)=\boxed{16}$
$a_4=16(2)=\boxed{32}$

(b) $f(n)=f(n-1)\cdot \frac{1}{3}$ with $f(1)=9$

$f(1)=9$
$f(2)=9\left(\frac{1}{3}\right)=\boxed{3}$
$f(3)=3\left(\frac{1}{3}\right)=\boxed{1}$
$f(4)=1\left(\frac{1}{3}\right)=\boxed{\frac{1}{3}}$

(c) $t_n=t_{n-1}\cdot\sqrt{2}$ with $t_1=3\sqrt{2}$

$t_1=3\sqrt{2}$
$t_2=3\sqrt{2}(\sqrt{2})=6$
$t_3=6(\sqrt{2})=6\sqrt{2}$
$t_4=6\sqrt{2}(\sqrt{2})=12$

$\boxed{6,\ 6\sqrt{2},\ 12}$

Exercise #6: Consider $a_1 = 2$ and $a_n = a_{n-1} \cdot 3$.

$r = 3$

(a) Generate the value of a_4.

$a_1 = 2$

$a_2 = 2(3) = 6$

$a_3 = 6(3) = 18$

$a_4 = 18(3) = \boxed{54}$

(b) How many times did you need to multiply 2 by 3 in order to find a_4.

3 times (1 less than the term #)

(c) Determine the value of a_{10}.

$a_{10} = 2 \cdot \underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$

$a_{10} = 2 \cdot 3^9$

$a_{10} = \boxed{39366}$

(d) Write a formula for the n^{th} term of a geometric sequence, a_n, based on the first term, a_1, r and n.

$a_n = a_1 \cdot (r)^{n-1}$

Term Now

First Term

Common ratio

Term # now

HW: Fluency section
#7-10